EUROPEAN TRANSACTIONS ON TELECOMMUNICATIONS
Eur. Trans. Telecomms. 2011; **22**:515–521
Published online 18 August 2011 in Wiley Online Library (wileyonlinelibrary.com). DOI: 10.1002/ett.1501

RESEARCH ARTICLE

On cost-sharing mechanisms in cognitive radio networks

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ABSTRACT

We present a framework and pertinent formulations for a coalition of secondary cognitive radios that are willing to lease inactive spectrum band from a primary system through auctioning and to share the received bandwidth and the associated cost among themselves using multiple access techniques. We cast this scenario to submodular class of games and show how a link can be established between the truthful auctioning mechanism and the cost-sharing algorithm. Simulation results verify that the deployed cost-sharing technique leads to encouraging the secondary cognitive radios to truthfully announce their bids. Copyright © 2011 John Wiley & Sons, Ltd.

KEY WORDS
cognitive radio networks; cost-sharing; throughput

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Received 23 June 2010; Revised 4 April 2011; Accepted 7 June 2011

1. INTRODUCTION

Although considerable research has been devoted to the applications of cooperative game theory [1–3] as well as optimisation methods (e.g. [5]) for modeling and analysis of cognitive radio (CR) systems, rather less attention is paid to the analysis of cost-sharing among an alliance of CRs as a spectrum rental strategy. This, for instance, can occur when, as shown in Figure 1, a service provider (either the primary or a secondary base station) provides a group of CR receivers with their required service and divides the whole charge among them. This finds various applications, for example, in ad hoc networks or mesh networks. As investigated in the IEEE 802.22 standard [4], the primary base station (e.g. TV broadcast) and primary users might exist in the same geographical vicinity of a network of secondary CRs as in Figure 1. For efficient secondary opportunistic spectrum access of primary idle bands or white spaces, the secondary base station coordinates resource allocation to the secondary CR network.

In [9], a game-theoretic model to obtain the optimal pricing for dynamic spectrum sharing in CR networks for environments where multiple primary services compete with each other to offer spectrum to secondary users has been presented, using Bertrand game and Nash equilibrium. In[10], a pricing model for short-term sublease of unutilised spectrum bands to different service providers, aiming at calculating the unit band prices that maximize the net profit of license holders while simultaneously satisfying buyers, has been proposed using noncooperative games. However, to the best of our knowledge, no previous work has studied cost-sharing mechanisms in CR networks. In other words, our work is distinct in that we use a cooperative game framework to investigate how the primary can allocate idle bands to bidding secondary CRs and divide the relevant cost among them, such that truthfulness is implemented as a dominant strategy for secondary CRs in announcing their bid amounts and also having incentives to cooperate with other network nodes than acting separately.

The primary can use auctioning to lease the band to a subset of secondary CRs that can afford the associated costs, which can include synchronisation, processor speed spent on traffic routing (which is proportional to traffic load) and methods to overcome interference. We aim at suggesting a fair spectrum allocation and cost-sharing mechanism among secondary CRs in the network. We analyse how an established coalition among a number of secondary CRs, subleasing a band from a primary user, can sustain by providing enough incentives for each CR not to abandon the cooperation with other secondary CRs. To this

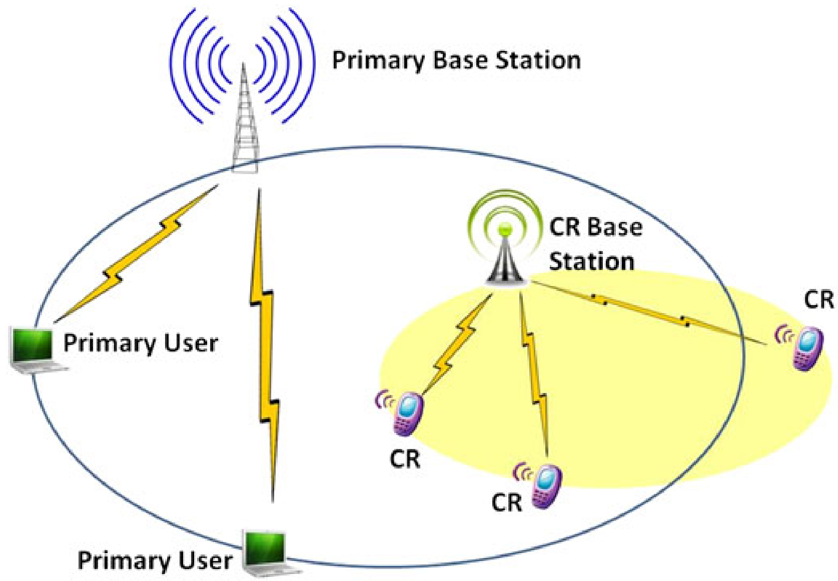

Figure 1. Primary providing unlicensed secondary cognitive radios with their spare spectral band.

end, we describe the main property of anticollusion cost-sharing strategies that is cross-monotonicity and propose a cross-monotonic cost-sharing scheme with decreasing differences, upon which a group-strategyproof mechanism can be established.

The organisation of this paper is as follows. Section 2 contains basic definitions and notions. In Section 3, we demonstrate and explore the deployed cost-sharing scheme and mechanism in detail. Simulation results and conclusions are presented in Sections 4 and 5, respectively.

2. NOTATION AND BASIC DEFINITIONS

In this section, we define basic notions from cooperative game theory used as tools to develop our analysis in Section 3.

Consider a set of n secondary CRs denoted by A[‡] that can form subsets $S \subseteq A$ to share the resources they lease from the primary user among themselves through multiple access techniques, for example carrier sense multiple access with collision avoidance and frequency, code or time division multiple access (FDMA, CDMA or TDMA, respectively). If the secondary CRs in $S \subseteq A$ are willing to divide the cost of spectrum usage in any arbitrary way, specified by a function $c(S)$ that indicates the cost of coalition S, that is $c : 2^{|A|} \to \mathbb{R}$, they are said to form a transferable utility (TU) cooperative game. $C(S)$ or the set of all possible outcomes for this CR TU game is composed of vectors:

$$x \in \mathbb{R}^{|S|} : \sum_{i \in S} x_i \leq c(S) \tag{1}$$

If an $|A|$ dimensional vector α satisfies the *budget balance*, that is

$$\sum_{j \in A} \alpha_j = c(A) \tag{2}$$

and *core property*, that is

$$\forall S \subseteq A, \sum_{j \in S} \alpha_j \leq c(S) \tag{3}$$

conditions, it is said to be in the *core* of the TU cooperative game (A, c). In other words, the notion of the core captures a stable case for which it is to the benefit of all secondary CRs to remain in the coalition.

The Bondareva–Shapley theorem [7, 8] gives necessary and sufficient conditions for the core of a game to be nonempty.

A vector λ that assigns a nonnegative weight λ_S to each subset $S \subseteq A$ is called a balanced collection of weights if $\sum_{S: j \in S} \lambda_S = 1, \forall j \in S$.

***Bondareva–Shapley theorem*:** A cost-sharing game (A, c) with transferable utilities has a nonempty core if and only if for every balanced collection of weights λ, we have $\sum_{S \in A} \lambda_S c(S) \geq c(A)$.

Proof. The solution of the linear program (4) being precisely $c(A)$ is the necessary and sufficient condition for nonemptiness of the core of the game (A, c):

$$\begin{aligned} &\text{Maximize} \quad \sum_{j \in A} \alpha_j \\ &\text{Subject to} \quad \forall S \subseteq A : \sum_{j \in S} \alpha_j \leq c(S) \end{aligned} \tag{4}$$

‡Notation used in this paper are mainly adapted from [6, 12].

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DOI: 10.1002/ett

By forming the dual of this linear program, we will have

$$\begin{aligned}&\text{Minimize} \quad \sum_{S\subseteq A} \lambda_S c(S)\\&\text{Subject to} \quad \forall j \in A: \sum_{S:j\in S} \lambda_S = 1, \forall S \subseteq A: \lambda_S \geq 0\end{aligned} \tag{5}$$

The solution of linear program (5) being equal to $c(A)$ is a necessary and sufficient condition for nonemptiness of the core. In other words, the core is nonempty if and only if $\sum_{S\subseteq A} \lambda_S c(S) \geq c(A)$. □

However, because in practice the core of many cost-sharing problems is empty [6], we consider an approximate core. A vector $\alpha \in R^A$ is in the γ core of (A, c) game if it satisfies the following:

$$\gamma\text{-budget balance} \quad \gamma c(A) \leq \sum_{j\in A} \alpha_j \leq c(A) \text{and} \tag{6}$$

$$\text{core property} \quad \sum_{j\in S} \alpha_j \leq c(S), \forall S \subseteq A \tag{7}$$

The necessary and sufficient condition for nonemptiness of γ core of a TU game (A, c) is $\sum_{S\subseteq A} \lambda_S c(S) \geq \gamma c(A), \forall\lambda$, where λ denotes balanced collection of weights.

3. COST-SHARING AND GROUP-STRATEGYPROOF MECHANISMS

3.1. Problem modeling and formulation

We associate the cost-sharing problem defined in Section 2 to group-strategyproof auctioning mechanisms. Cost-sharing models the pricing problem for a primary with a given set of secondary CRs. The primary conducts an auction to select a subset of the set of secondary CRs on the basis of their bids and the cost. We aim at designing an auction in which the group of secondary CRs are encouraged to bid truthfully.

Let A be a set of n secondary CRs interested in leasing the spectrum band from the primary. For the primary, the cost of providing the secondary CRs with their required bandwidth is a function $c: 2^{|A|} \to \mathbb{R}^+ \bigcup\{0\}$. As defined in Section 2, for a subset of secondary CRs, $S \subseteq A$, the cost of providing service by the primary is denoted by $c(S)$. Each secondary CRi has a maximum valuation of $u_i \in \mathbb{R}$ for the band. For instance, the valuation of each CR for each of M available sub-bands k can be associated to the channel throughput or ergodic capacity, that is

$$w_i = \alpha \log\left(1 + \frac{P_{ik} g_{ik}}{\sigma_{ik}^2}\right) \tag{8}$$

where P_{ik} is the transmission power of CRi over subchannel k, α is a proportionality constant to the monetary cost and σ_{ik}^2 is the noise power on subchannel k if used by CRi. The value of channel gain g_{ik} can be known to each CR immediately before disclosing the bid if the secondary CRs are allowed to carry out a quick channel sounding to measure channel quality from their standpoint.

We define a binary indicator variable q_i for each secondary CRi as

$$q_i = \begin{cases} 1 & \text{if CR}i \text{ receives the band} \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

If secondary CRi is allocated to receive a band, its utility is given by $u_i q_i - x_i$, where x_i is the amount this CR has to pay when allocated the spectrum. Otherwise, its utility is zero. The primary receives the vector $\mathbf{b}$ of bids b_i from secondary CRs and selects a subset of them $Q(\mathbf{b}) \subseteq A$ to lease the band to and assigns a vector of payments $\mathbf{p}(\mathbf{b}) \in \mathbb{R}^n$. We assume that the allocation and charging scheme have the following properties [12]:

- *No positive transfer*: $p_i \geq 0$, (p_i is the payment of CRi.
- *Voluntary participation*: A secondary is not charged if it is not being allocated with any spectral band. On the other hand, if the primary allocates the band to a secondary CR$i \in Q(\mathbf{b})$, the payment for this CR should not exceed its bid, that is $p_i \leq b_i$.
- *Consumer sovereignty*: For each secondary, there is some bid amount for which it receives the bandwidth, no matter what other secondary CRs' bids are.

Furthermore, we are looking for allocation and payment mechanisms with the following features:

- *γ-budget-balanced* with respect to cost function c; that is, the total amount the mechanism charging the secondary CRs $\sum_{i\in Q(\mathbf{b})} x_i$ is between $\gamma c(Q(\mathbf{b}))$ and $c(Q(\mathbf{b}))$.
- *Group-strategyproof* that is mainly associated with truthfulness of the bidding secondary CRs to announce their true bids. To be group-strategyproof, in addition to no positive transfer, voluntary participation and consumer sovereignty properties mentioned previously, the mechanism should satisfy the following [12]: for a coalition $S \subseteq A$ of secondary CRs and two vectors of truthful bids $\mathbf{u}$ and nontruthful bids $\mathbf{u}'$ satisfying $u_i = u_i'$ for every $i \notin S$, if $(Q, \mathbf{p})$ and $(Q', \mathbf{p}')$ denote the outputs of the mechanism when the bids are $\mathbf{u}$ and $\mathbf{u}'$, respectively, a mechanism is *group-strategyproof* when for every coalition S of secondary CRs, if the inequality $u_i q_i' - p_i' \geq u_i q_i - p_i$ holds, then it holds with equality for every $i \in S$. In other words, there should not be any other coalition S and vector $\mathbf{u}'$ of bids such that if CRs in S announce $\mathbf{u}'$ instead of $\mathbf{u}$ (their true value) as their bids, then

every secondary CR of the coalition S at least benefits as in the truthful case, and at least one secondary receives more payoff.

According to a theorem by Moulin [12], cost-sharing methods satisfying *cross-monotonicity* property, in addition to the above-mentioned properties, can be used to design group-strategyproof cost-sharing mechanisms. Cross-monotonicity implies that the secondary CRs should not lose utility when more nodes join the coalition.

A cost-sharing scheme is a function for the cost sharing game (A, c) that for each subset $S \subseteq A$ assigns a cost allocation for S. In other words,

$$\xi : A \times 2^{|A|} \to \mathbb{R}^+$$
$$\forall S \subseteq A, i \notin S, \xi(i, S) = 0 \qquad (10)$$

The following characterises a cross-monotonic cost-sharing scheme:

$$\xi(i, S) \geq \xi(i, S \bigcup T), \quad \forall S, T \subseteq A \text{ and } \forall i \in S \qquad (11)$$

Given a cross-monotonic cost-sharing scheme ξ for the cost-sharing game (A, c), a cost-sharing mechanism as in Figure 2 has been defined [12] that is group-strategyproof and γ-budget balanced if ξ is a γ-budget-balanced cross-monotonic cost-sharing scheme [13].

For positive real values (including 0) of bids, this mechanism yields the unique maximal set $S \subseteq A$ for which the bids of all secondary nodes exceed their assigned cost. The type of bidding misreport that can occur is underbidding, because for an overbidding CR, $b'_i > \xi(i, S) > b_i$, for S denoting the set of winners when CRi's bid is b'_i that is greater than its true bid b_i; this implies that CRi has to pay a higher value than the actual one, defeating the purpose of collusion that was meant to yield more payoff.

Furthermore, let $\mathcal{F}$ denote the set of all idle frequency bands (e.g. TV white spaces) available for secondary spectrum access through the primary base station. In other words, $\mathcal{F}$ is the subset of licensed frequency bands not used by the primary network in a specific time/space interval and therefore available to be deployed by secondary CR network. In addition, we assume that through a centralised mechanism, for example by a cognitive base station, the set $\mathcal{F}$ is partitioned into M equal bandwidth subchannels [11] for use by secondary CRs. Each subchannel is allocated to not more than one CR, and each CR is assumed to use not more than one subchannel. The channel state information is assumed known at the CRs through channel sounding or use of pilot signals.

In the following, we present the details of the spectrum sharing configuration and propose a cross-monotonic cost-sharing scheme that results in a group-strategyproof cost-sharing mechanism.

3.2. Cost-sharing scheme

Our solution consists of two main stages. First, we propose the optimal policy the service provider to secondary CRs (either primary itself or a secondary base station) can adopt to decide which CRs to include in its network to receive the band. Next, we show how the service provider can divide the total incurred cost among selected CRs in stage one in the optimal manner, such that they have no incentive to misreport their bids or disjoin the coalition formed in the previous stage. Without loss of generality and specifically for independent identically distributed subchannels, we propose the cross-monotonic cost-sharing scheme to be equal to the weighted division of cost of each set $S \subseteq A$, as shown in Figure 3, among the total number of elements in the set:

$$\xi(i, S) = \begin{cases} \frac{\delta(i)c(S)}{|S|} & \text{if } i \in S, \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

The weight of each CRi is denoted by $\delta(i)$ such that $\sum_{i \in S} \delta(i) = 1$ and is proportional to the potential throughput estimation of each CR through channel sounding. This cost function has decreasing differences, thus encouraging the CRs to join the grand coalition. In Figure 3, d is a constant to give $c(A) > 0$.

4. SIMULATION RESULTS

In this section, we show how, by using our proposed cost-sharing scheme as in Figure 3, the mechanism in Figure 2

Initialize $S \leftarrow A$.
Repeat
Let $S \leftarrow \{i \in S : b_i \geq \xi(i, S)\}$.
Until $\forall i \in S, b_i \geq \xi(i, S)$.
Return $Q = S$ and $p_i = \xi(i, S)\ \forall i$.

Figure 2. Cost-sharing mechanism.

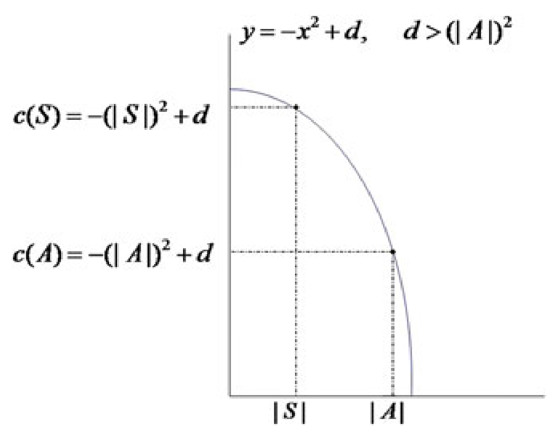

Figure 3. Proposed cost function with decreasing differences.

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DOI: 10.1002/ett

elicits truthful bids from secondary CRs and eliminates incentives for underbidding.

Figure 4 shows that our proposed cost-sharing scheme leads to a mechanism that reduces the chances of receiving the bandwidth for secondary CRs that do not announce their true bids by more than 40%. We carried out the simulation for a total number of 30 CRs that can, for instance, represent the number of unlicensed users in an office building trying to gain WiFi access. In the first case, all of them are truthful in declaring their valuations. In the second case, half of them underbid. The winning chances for colluding CRs in 100 iterations are shown in Figure 4 for both cases.

Furthermore, Figure 5 depicts how this method alleviates collusion among secondary CRs, in the auction carried out by the primary, through decreasing the utility or payoff of cheating CRs. The utility is defined as $u_i q_i - x_i$ for $q_i = 1$ (and zero when $q_i = 0$), where u_i is CRi's valuation and x_i is its share of cost returned by the mechanism. Simulations for Figures 4 and 5 were carried out by random generation of bids in both truthful and underbidding cases. Figure 5 shows the sum of utilities for the winners among the cheating half of CRs, for the two cases of bidding truthfully and nontruthfully. As evident in Figure 5, the utility of each CR drops if commits a bidding misreport. The total number of CRs varies from 20 to 35, and half of them collude by underbidding. The sum of the utilities of the cheating half is shown for both cases, normalised by the total utility in the nontruthful scenario. Therefore, even if nontruthful CRs win the auction, their actual utility is less than the truthful case.

Figures 6 and 7 compare utilities of the same winning CRs for the two cases of being truthful, that is Figure 6, and nontruthful, that is Figure 7. Here, nontruthful CRs comprise 30% to 70% of a range of total CRs from 20 to 35. As these figures show, underbidding the secondary

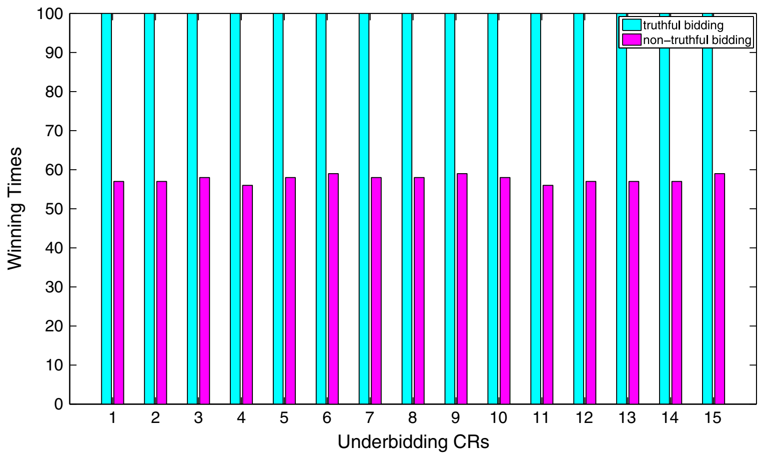

Figure 4. Winning chances of secondary cognitive radios for truthful and nontruthful bidding.

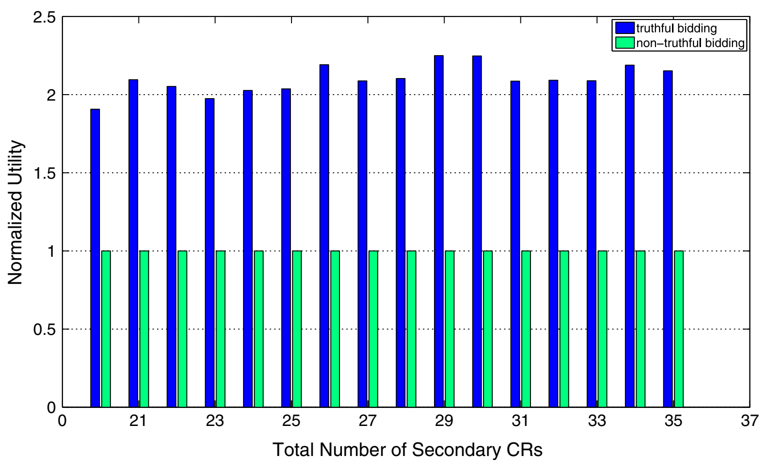

Figure 5. Sum of underbidding winning cognitive radios utilities for truthful and nontruthful cases normalised to total utility of nontruthful bidding.

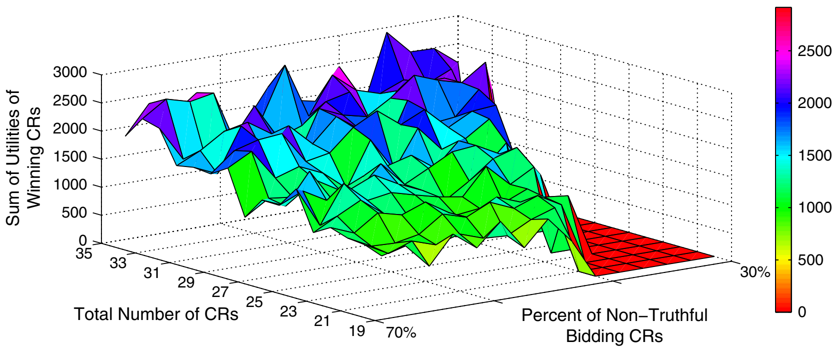

Figure 6. Sum of utilities of winning cognitive radios.

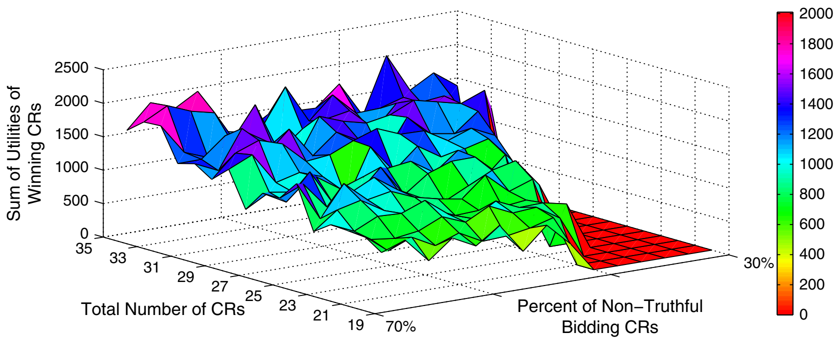

Figure 7. Sum of utilities of nontruthful winning cognitive radios.

CRs actually decrease their utilities. To illustrate the efficiency of our proposed mechanism, the utility of same winning CRs in Figure 7 is normalised by their utility in Figure 6 to give Figure 8. As Figure 8 shows, the utility of underbidding winning CRs is always less than their utility when they bid truthfully, regardless of the total number of CRs and the percentage of nontruthful CRs. In fact, it does not exceed 85% of the truthful utility at maximum.

5. CONCLUSIONS

We investigated a scenario where a network consisting of secondary CR nodes subleases idle bands from the primary system. We put forward a model for fairly dividing the costs relevant to secondary CR nodes' multiple access to the bandwidth. For this purpose, we proposed a cost-sharing scheme with decreasing differences upon which the cost-sharing mechanism in Figure 2 can be established.

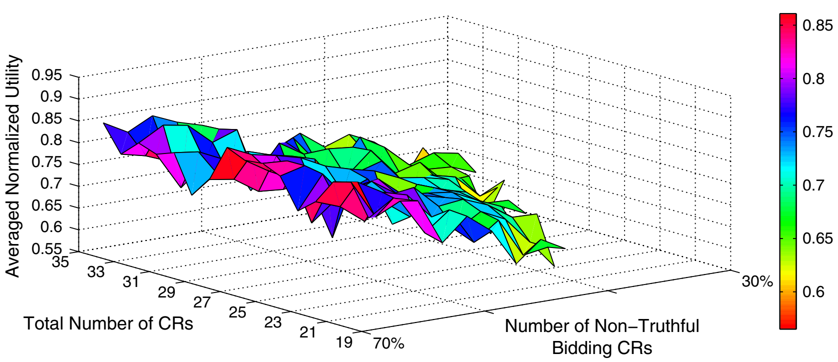

Figure 8. Sum of utilities of nontruthful winning cognitive radios in Figure 7 normalised to sum of utilities of same cognitive radios in the truthful case as in Figure 6.

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DOI: 10.1002/ett

The deployed mechanism leads to a group-strategyproof property; that is, the cost of receiving spectral band is divided in such a way among seconary CRs that prevents them from nontruthful bidding in the auction held by the primary system.

Our simulations verified the anticollusion characteristic of this scheme through decreasing both winning chances and utilities of nontruthful CRs. More specifically, simulations for secondary CRs, representing unlicensed users in an office building trying to gain WiFi access, show this algorithm reduces the winning chance of nontruthful CRs by more than 40% and their utility to an average of 70% of the truthful case. These facts result in successful coalition formation by secondary service provider that can either be the primary or the secondary base station.

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